Unit-III IMPACT (Effect)
Influence

Impulsive force

A large force which acting on a body for an infinitesimally small period, produces a finite change of momentum in that interval, is called an impulsive force

Ex:

The force experienced by a ball due to hit by a bat

Impulse: The effect of the action of an impulse force is measured by the change in momentum produced by the force.

This change is called the impulse of the impulsive action.

$\boxed{F = MA}$ — F: force, M: mass, A: acceleration

$F = m(v - u)$ — v: after, u: before

Equation of an impulse imparted to a particle of mass m

$$I (= F) = MA$$
$$= m(\bar{v} - \bar{u})$$
$$= m\bar{v} - m\bar{u}$$
$$= \text{change of momentum}$$

where $\bar{F}$ - constant force acting on a particle.

m - mass of the particle

A - Acceleration of the particle

t - Time

$\bar{u}$ & $\bar{v}$ - velocity at before and after the impulsive action.

If 't' is the short time during which the impulsive force acts, then

$$\bar{I} = m\bar{v} - m\bar{u} = [m\bar{v}]_0^t = \int_0^t m\,d\bar{v} = \int_0^t m\frac{d\bar{v}}{dt}\,dt$$
$$= \int_0^t mA\,dt = \int_0^t F\,dt$$

(Impulse)

Note:

Impulse → It is measured by the unit of momentum

→ but not by the unit of force.

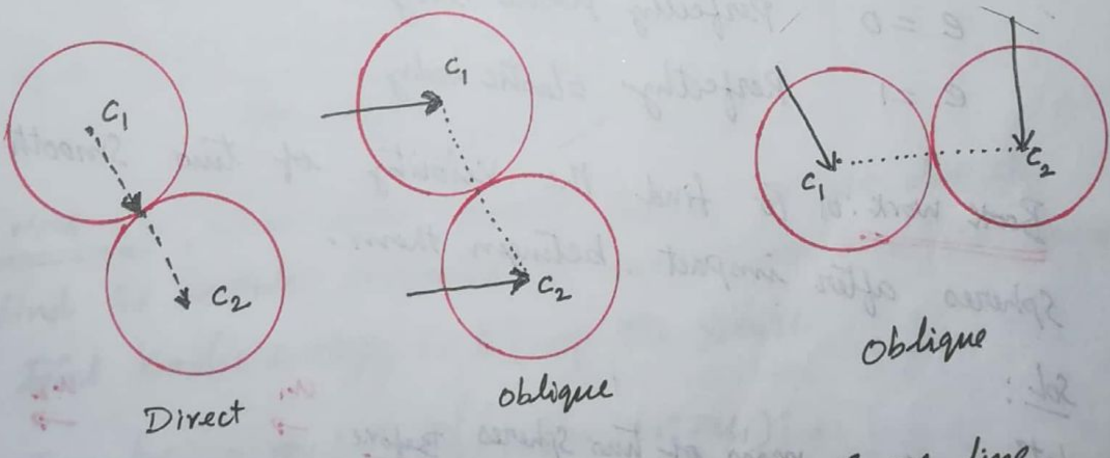
Impact of two Smooth Spheres

Suppose two smooth spheres collide with each other.

Impact of Spheres

Direct impacts | oblique impacts

If C_1 & C_2 are the positions of the centres of the spheres at the time of impact and if the centres of the spheres had been moving before the impact along the straight line through C_1 and C_2, then the impact is said to be direct impact; otherwise, it is said to be oblique impact.



[* when two spheres, moving along the same line, collide the impact is called direct impact

* when two spheres, moving along different line, collide the impact is called oblique impact]

Conservation of linear momentum:

Total momentum before impact = Total momentum after Impact

$$\boxed{m_2 v_2 + m_1 v_1 = m_2 u_2 + m_1 u_1}$$

Momentum = mass × velocity

Newton's Experimental law:

[Relative velocity of B with respect to A After impact] = − e [Relative velocity of B with respect to A Before impact]

$$\boxed{v_2 - v_1 = -e\,(u_2 - u_1)}$$

where,

e – coefficient of elasticity or Restitution

lies between 0 and 1

$e = 0$ Perfectly plastic body

$e = 1$ Perfectly Elastic body

Book work: 01 To find the velocity of two smooth Spheres after impact between them.

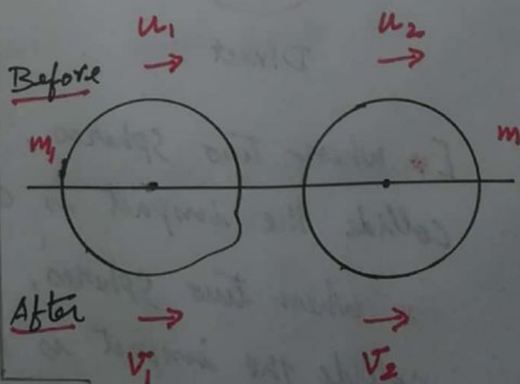
Sol:

Let m_1, m_2 – mass of two spheres

u_1, u_2 – velocity of two spheres before impact. $(u_1 > u_2)$

v_1, v_2 – velocity of two spheres after impact

e – coefficient of restitution



By the principle of Conservation of linear momentum

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 \longrightarrow (1)$$

By Newton's Experimental law,

$$v_2 - v_1 = -e(u_2 - u_1)$$

$$v_2 - v_1 = -eu_2 + eu_1 \longrightarrow (2)$$

$$(1) \Rightarrow \quad m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$(2)\times m_1 \Rightarrow \quad m_1 v_2 - m_1 v_1 = -m_1 e u_2 + m_1 e u_1$$

$$(m_1+m_2) v_2 = m_1 u_1 + m_1 e u_1 + m_2 u_2 - m_1 e u_2$$

$$= u_1 m_1 (1+e) + u_2 (m_2 - m_1 e)$$

$$v_2 = \frac{m_1 u_1 (1+e) + u_2 (m_2 - e m_1)}{m_1 + m_2}$$

$$(1) \Rightarrow \quad m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$(2)\times m_2 \Rightarrow \quad m_2 v_2 - m_2 v_1 = -m_2 e u_2 + m_2 e u_1$$

$$v_1 (m_1 + m_2) = m_1 u_1 - m_2 e u_1 + m_2 u_2 + m_2 e u_2$$

$$v_1 = \frac{u_1 (m_1 - m_2 e) + m_2 u_2 (1+e)}{m_1 + m_2}$$

Book Work : 02 when two smooth sphere collide directly to find the impulse imparted to each sphere and the change in total Kinetic energy (K.E) of the sphere.

Sol. Impulse $= m_2 (v_2 - u_2) = -m_1 (v_1 - u_1)$

$$I = -m_1 (v_1 - u_1)$$

$$= -m_1 \left(\frac{u_1 (m_1 - m_2 e) + m_2 u_2 (1+e)}{m_1 + m_2} - u_1 \right)$$

$$= -m_1 \left(\frac{u_1 m_1 - u_1 m_2 e + m_2 u_2 + m_2 u_2 e - u_1 m_1 - u_1 m_2}{m_1 + m_2} \right)$$

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$$= \frac{-m_1}{m_1+m_2}\left[m_2u_2(1+e) - u_1m_2(1+e)\right]$$

$$= \frac{-m_1}{m_1+m_2}\left[(1+e)(m_2u_2 - u_1m_2)\right]$$

$$= \frac{-m_1}{m_1+m_2}\left[m_2(1+e)(u_2-u_1)\right]$$

$$I = \frac{m_1m_2(1+e)(u_1-u_2)}{m_1+m_2}$$

$$\left.\begin{array}{l}\text{Increase in K.E (or)}\\ \text{Loss of K.E}\end{array}\right\} = \text{Total K.E after impact} - \text{Total K.E before Impact.}$$

$$= \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right) - \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right)$$

$$= \frac{1}{2}\left[m_1(v_1^2-u_1^2) + m_2(v_2^2-u_2^2)\right]$$

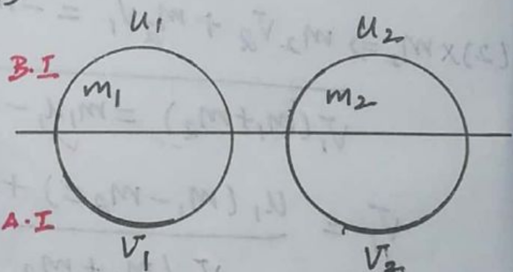
$$= \frac{1}{2}\left[m_1(v_1-u_1)(v_1+u_1) + m_2(v_2-u_2)(v_2+u_2)\right] \longrightarrow (1)$$

W.K.T

$$I = m_2(v_2-u_2) = -m_1(v_1-u_1)$$

Newton's Experimental law,

$$v_2 - v_1 = -e(u_2-u_1)$$



$$(1) \Rightarrow \quad = \frac{1}{2}\left[-(v_1+u_1)I + (v_2+u_2)I\right]$$

$$= \frac{I}{2}\left[-v_1-u_1+v_2+u_2\right]$$

$$= \frac{I}{2}\left[v_2-v_1+u_2-u_1\right]$$

$$= \frac{I}{2}\left[-e(u_2-u_1) + (u_2-u_1)\right]$$

$$= \frac{I}{2}(u_2-u_1)(1-e)$$

Sub. 'I' value

$$\left.\begin{array}{l}\text{Increase in}\\ \text{K.E (or)}\\ \text{loss of K.E}\end{array}\right\} = \frac{m_1m_2}{2(m_1+m_2)}(u_1-u_2)(1+e)(1-e)(u_2-u_1)$$

$$= \frac{-m_1m_2}{2(m_1+m_2)}(u_1-u_2)^2(1-e^2) \qquad \left(a^2-b^2 = (a+b)(a-b)\right)$$

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3) To show that when two spheres of equal masses 'm' collide directly the velocities of the sphere are interchange if $e=1$.

Sol. Given $e=1$

By principal of conservation of momentum

$$mv_1 + mv_2 = mu_1 + mu_2$$

$\div$ by m, $v_1 + v_2 = u_1 + u_2 \longrightarrow (1)$

By Newton's Experimental law

$$v_2 - v_1 = -e(u_2 - u_1) \qquad (\because e=1)$$

$$v_2 - v_1 = -u_2 + u_1 \longrightarrow (2)$$

(1) $\Rightarrow$ $\cancel{v_1} + v_2 = u_1 + \cancel{u_2}$

(2) $\Rightarrow$ $\cancel{-v_1} + v_2 = u_1 - \cancel{u_2}$

$$2v_2 = 2u_1$$

$$\boxed{v_2 = u_1}$$

(1) $\Rightarrow$ $v_1 + \cancel{v_2} = \cancel{u_1} + u_2$

(2) $\Rightarrow$ $-v_1 + \cancel{v_2} = \cancel{u_1} - u_2$

$$2v_1 = 2u_2$$

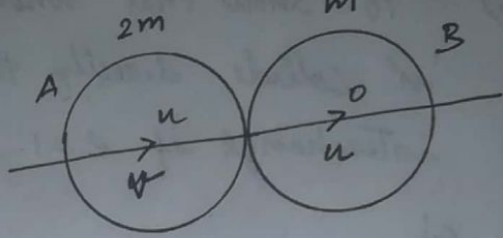
$$\boxed{v_1 = u_2}$$

$\therefore$ The velocities are interchanged

4) A ball of mass '2m' impinges directly on a ball of mass 'm' which is at rest. If the velocity after impact of the latter ball is equal to that of the former before impact S.T $e = \frac{1}{2}$

Sol.

	A	B
mass	2m	m
Before Impact	u	0
After Impact	v	u



By Principle of conservation of linear momentum

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$2m(v) + mu = 2mu + m(0)$$

$$2mv + mu = 2mu$$

$\div$ by m

$$2v + u = 2u$$

$$2v = 2u \longrightarrow (1)$$

Newton's Experimental law

$$v_2 - v_1 = -e(u_2 - u_1)$$

$$u - v = -e(0 - u) \qquad (\because u = 2v)$$

$$2v - v = -e(-2v)$$

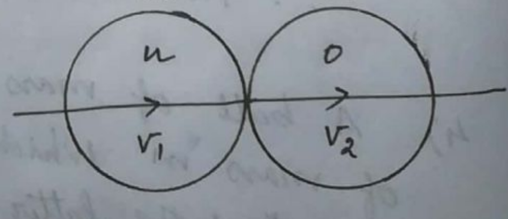
$$\cancel{v} = 2\cancel{v}e$$

$$\boxed{e = \frac{1}{2}}$$

5) A ball 'A' impinges directly on an exactly equal mass and Similar ball B lying on a Smooth horizontal table if e is the coefficient of restitution. P.T after impact the velocity of B is to that of A is $1+e : 1-e$.

Sol.

	A	B
mass	m	m
B.I	u	0
A.I	v_1	v_2



To Prove

$$\frac{V_2}{V_1} = \frac{1+e}{1-e}$$

By principle of Conservation of momentum

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$m v_1 + m v_2 = m u + m(0)$$

$\div$ by m $\quad v_1 + v_2 = u \longrightarrow (1)$

By Newton's Experimental law

$$v_2 - v_1 = -e(u_2 - u_1)$$

$$v_2 - v_1 = -e(0 - u)$$

$$v_2 - v_1 = eu \longrightarrow (2)$$

$(1)+(2) \Rightarrow$

$$v_1 + v_2 + v_2 - v_1 = u + eu$$

$$2v_2 = u(1+e) \longrightarrow (3)$$

$(1)-(2) \Rightarrow$

$$v_1 + \cancel{v_2} - \cancel{v_2} + v_1 = u - eu$$

$$2v_1 = u(1-e) \longrightarrow (4)$$

$\frac{(3)}{(4)} \Rightarrow \frac{2v_2}{2v_1} = \frac{u(1+e)}{u(1-e)}$

$$\frac{v_2}{v_1} = \frac{1+e}{1-e}$$

$$\therefore v_2 : v_1 = 1+e : 1-e$$

6) Two spheres A and B of same size lie on a smooth horizontal circular groove at opposite ends of a diameter. A is projected along the groove and after a time 't' impinges upon B. Show that a second impact will occur after a time $2t/e$. where 'e' is coefficient of restitution.

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Sol.:

Let the velocity of A is u

W.K.T

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{time} = \frac{\text{distance}}{\text{Speed}}$$

$$t = \frac{\pi a}{u} \quad (\because \text{circumference of Semicircular} = \pi . r \text{ where } r = a$$

After impact of the velocity of A is V_1 and the velocity of B is V_2

By Newton's Experimental law,

$$V_2 - V_1 = -e(u_2 - u_1)$$

$$V_2 - V_1 = -e(0 - u)$$

$$V_2 - V_1 = eu$$

i.e., Relative Speed $= eu$

The relative distance it has to travel before the Second impact is $2\pi a$

$$\text{time} = \frac{\text{distance}}{\text{Speed}}$$

$$= \frac{2\pi a}{eu} = \frac{2\cancel{u}t}{e\cancel{u}} \quad (\because \pi a = ut$$

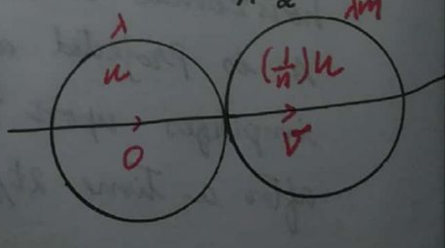
$$\text{time} = \frac{2t}{e}$$

---x---

7) A ball overtaken another ball 'm' times its mass. which is moving with $(\frac{1}{n})^{th}$ of its velocity in the Same direction if the impact reduces the first ball at rest. P.T Coefficient of elasticity is $e = \frac{m+n}{m(n-1)}$ deduce that $m > \frac{n}{n-2}$

Sol

	A		B
Mass	λ	-	λm
B.I	u	-	$\frac{u}{n}$
A.I	0	-	v



Let, A, B - Two balls

$\lambda, \lambda m$ - mass of A, B

$u, \frac{u}{n}$ - velocities of A, B before impact

$0, v$ - velocities of A, B after impact

By principle of conservation of linear momentum

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$\lambda(0) + \lambda m(v) = \lambda u + \lambda m\left(\frac{u}{n}\right)$$

$$mv = u + \frac{mu}{n} \longrightarrow (1)$$

By Newton's Experimental law,

$$v_2 - v_1 = -e(u_2 - u_1)$$

$$v - 0 = -e\left(\frac{u}{n} - u\right)$$

$$v = -e\left(\frac{u}{n} - u\right) \longrightarrow (2)$$

Sub. (2) in (1).

$$m\left(-e\left(\frac{u}{n} - u\right)\right) = u + \frac{mu}{n}$$

$$-em\cancel{u}\left(\frac{1}{n} - 1\right) = \cancel{u}\left(1 + \frac{m}{n}\right)$$

$$-em\left(\frac{1-n}{\cancel{n}}\right) = \frac{m+n}{\cancel{n}}$$

$$e = \frac{m+n}{m(n-1)}$$

W.K.T, $0 < e < 1$

$e < 1$

$$\frac{m+n}{m(n-1)} < 1$$

$$m + n < m(n-1)$$

$$n < m(n-1) - m$$

$$n < m[n-1-1] \Rightarrow n < m(n-2)$$

$$\boxed{m > \frac{n}{n-2}}$$

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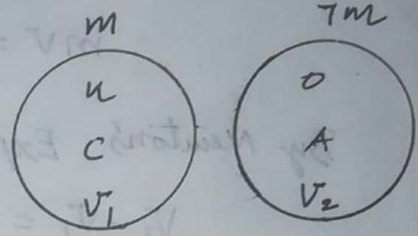
8) The mass of 3 Spheres A, B, C are 7m, 7m, m. The Coefficient of restitution $e=1$, then Centres are in straight line, c lies between A and B. A and B are at rest initially, c is given a velocity in the line of centres in the direction of A. Show that it strikes A twice and B once and the final velocities of A, B and C are in the ratio 21 : 12 : 1.

Sol.: Impact - I



	C	A
mass	m	7m
B.I	u	0
A.I	V_1	V_2

By Principal of conservation of linear momentum

$$m_1 V_1 + m_2 V_2 = m_1 u_1 + m_2 u_2$$

$$m_1 V_1 + 7m V_2 = mu + m(0)$$

÷ by m, $$V_1 + 7V_2 = u \longrightarrow (1)$$

By Newton's experimental law,

$$V_2 - V_1 = -e(u_2 - u_1)$$

$$V_2 - V_1 = -1(0-u)$$

$$V_2 - V_1 = u \longrightarrow (2)$$

(1) + (2) ⟹ $$8V_2 = 2u$$

$$\boxed{V_2 = u/4}$$

Sub V_2 in (2) we get

$$\frac{u}{4} - V_1 = u \implies V_1 = \frac{u}{4} - u$$

$$\boxed{V_1 = \frac{-3u}{4}}$$

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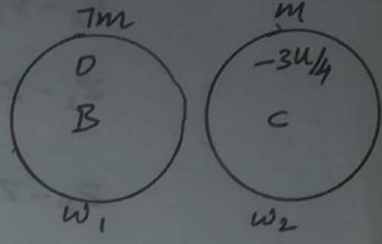
Impact-II

By principle of conservation of linear momentum



$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$7m w_1 + m w_2 = 7m(0) + m\left(-\frac{3u}{4}\right)$$

$$7w_1 + w_2 = \frac{-3u}{4} \longrightarrow (3)$$

	B	C
mass	7m	m
B.I	0	$\frac{-3u}{4}$
A.I	w_1	w_2

By Newton's Experimental law

$$v_2 - v_1 = -e(u_2 - u_1)$$

$$w_2 - w_1 = -1\left(-\frac{3u}{4} - 0\right)$$

$$w_2 - w_1 = \frac{3u}{4} \longrightarrow (4)$$

$(3) - (4) \Rightarrow$

$$7w_1 + w_2 - w_2 + w_1 = -\frac{3u}{4} - \frac{3u}{4}$$

$$8w_1 = -\frac{6u}{4}$$

$$\boxed{w_1 = \frac{-3u}{16}}$$

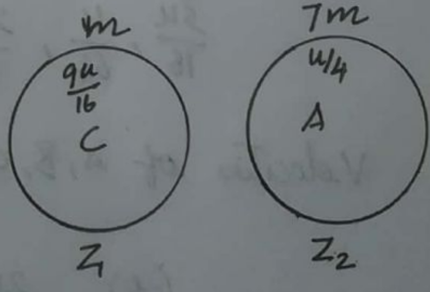
Sub. w_1 in (4), $\quad w_2 + \frac{3u}{16} = \frac{3u}{4}$

$$w_2 = -\frac{3u}{16} + \frac{3u}{4}$$

$$\boxed{w_2 = \frac{9u}{16}}$$

Impact-III

By Principal of conservation of Linear momentum



$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$m z_1 + 7m z_2 = m\left(\frac{9u}{16}\right) + 7m\left(\frac{u}{4}\right)$$

$\div$ by m, $\quad Z_1 + 7Z_2 = \frac{9u}{16} + \frac{7u}{4}$

$$z_1 + Z_2 = \frac{37u}{16} \longrightarrow (5)$$

	C	A
mass	m	7m
B.I	$9u/16$	$u/4$
A.I	Z_1	Z_2

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By Newton's Experimental law,

$$V_2 - V_1 = -e(u_2 - u_1)$$

$$z_2 - z_1 = -1\left(\frac{u}{4} - \frac{9u}{16}\right)$$

$$z_2 - z_1 = -\left(-\frac{5u}{16}\right)$$

$$z_2 - z_1 = \frac{5u}{16} \longrightarrow (6)$$

$$(5)+(6) \Rightarrow z_1 + 7z_2 + z_2 - z_1 = \frac{37u}{16} + \frac{5u}{16}$$

$$8z_2 = \frac{42u}{16}$$

$$\boxed{z_2 = \frac{21u}{64}}$$

Sub. z_2 in (6)

$$\frac{21u}{64} - z_1 = \frac{5u}{16}$$

$$-z_1 = \frac{5u}{16} - \frac{21u}{64}$$

$$-z_1 = \frac{20u - 21u}{64}$$

$$-z_1 = \frac{-u}{64}$$

$$\boxed{z_1 = \frac{u}{64}}$$

Velocities of B, C and A after impact are

$$\frac{3u}{16}, \frac{u}{64}, \frac{21u}{64}$$

Velocities of A, B, C is $\frac{21u}{64}, \frac{3u}{16}, \frac{u}{64}$

i.e; 21 : 12 : 1